Hilbert fractal curves for HTS miniaturized filters

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***Abstract* — This work presents novel compact High Temperature Superconductor (HTS) microstrip resonators based on Hilbert fractal curves. Their miniaturization performance has been investigated, emphasizing the parameters which allow to obtain a good trade off between compact size and quality factor. Hilbert resonators with different arrangements have been designed and assessed by using full wave simulators. In order to experimentally verify our analysis, a four pole quasi elliptic filter, in the cascade quadruplet configuration, with f_0 close to 2.45GHz, have been fabricated by patterning a 10x10mm^2 YBCO film on MgO substrates. Minimum insertion losses of the order of (0.1-0.2) dB have been measured.**

***Index Terms* — Microstrip filters, High-Temperature Superconductors, Fractals, Microwave devices, Mobile Communication.**

I. INTRODUCTION

In recent years, the miniaturization trend of planar filters has received a new and growing interest because of the discovery of HTS. The miniaturization causes the corresponding increase of current density in the strip. High current densities in conventional metallic strips give rise to a very large amount of dissipative losses with a dramatic and unacceptable worsening of filter performance. In contrast, the very low surface resistance of HTS films at microwave frequencies allows to fabricate high performance compact planar resonators and filters.

Nowadays, HTS filters seem to be one of the most adequated to satisfy the needs of the modern telecommunication systems, when maximum compactness of the microwave circuitry and still stringent filter performance are required [1]. In this context, new highly compact HTS resonators have been recently proposed and successfully tested [2]-[5]. Despite their different shapes, all these structures are based on the miniaturizing principle to fold the elementary straight line resonator in very sophisticated ways in order to fit it in a compact area. Considering this scenario, in the present paper we report the results, in terms of miniaturization and overall performance, obtained by using HTS filters shaped as Hilbert curves [6].

II. HILBERT MICROSTRIP RESONATORS

In 1892, in a study about the existence of special curves which present space filling capabilities and the property of being everywhere continuous, the German mathematician David Hilbert presented the sets of curves shown in Fig.1 for the first four iterations ($k=1..4$). As evidenced by the presence of the background grid, the Hilbert curve with $k=1$ connects the centres of the four parts in which is divided the original square. For $k=2$, the same criterion can be applied dividing the square in 16 parts and connecting the centres in the same way. For the general kth iteration, 2^{2k} divisions are realized and consequently the curve will be composed of (2^{2k}-1) segments, all with the same length.

Recently and accordingly to what just mentioned, the miniaturization performances by means of Hilbert curves in the fabrication of small antennas have been intensively investigated [7]-[9]. These studies have clearly shown that, increasing the iteration level while keeping fixed the external side (S, see Fig.1), the resonant frequencies of a Hilbert antenna lower whereas the radiation characteristics worsen by decreasing the radiation resistance. From Hilbert microstrip resonators point of view, this last property suggests their good performances in lowering the packaging losses due to the radiated field. Moreover, analyzing the data reported in literature, it can be observed that in every k case, the fundamental resonance frequency is however higher than the fundamental frequency of a $\lambda/4$ monopole with the same length. This phenomenon is due to the couplings between the different turns of the Hilbert resonator which practically define an equivalent shorter path for the current. Obviously this effect gets stronger with k increasing, since a reduction of the interspacing between turns (g) takes place. Thus, a saturation the structure miniaturization capability occurs for large values of k. In other words, the ratio $f_0(k+1)/f_0(k)$ between the resonant frequencies of two consecutive iterations tends rapidly towards 1, instead of 0.5, which should be, considering the almost doubling of the length. This means, from a practical point of view, that only the first iterations ($k \leq 5$ or 6) of a *Hilbert resonator guarantee an effective* miniaturization improvement.

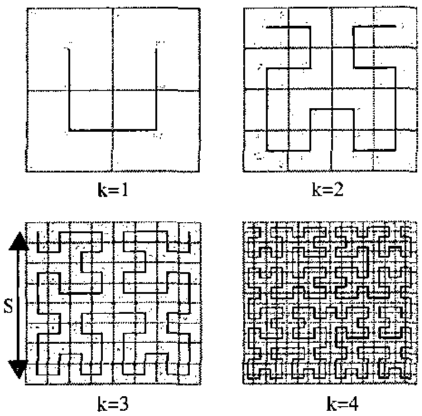


Fig. 1. Hilbert curves with different iteration levels k.

In Hilbert microstrip resonators, the strip width w and the interspacing between turns g are the parameters which define the trade off between miniaturization and quality factor Q. To obtain highly miniaturized resonators, which in our case means adopting curves with high k, the value of w is considerably narrowed. In turn, the current density increase, the dissipation losses increase and the quality factor decrease.

To illustrate this, Fig.2a shows some simulated results on how the resonant frequency f_0, depends on the external side S, as a function of Hilbert curve iteration k. In this simulation we have kept the ratio width of the strip and the interspacing between turns constant to 1 (i.e. w/g=1). On the other hand, Fig.2b reports the required w to fit a k Hilbert curve inside a square SxS, while keeping w/g=1.

From this preliminary analysis, the Hilbert resonator with $k=4$ and $w/g=1$ shown in Fig.3 seems to be the best candidate to assure a good trade-off between miniaturization level and w at f_0=2GHz. In this case, the strip width is 115 μm and the external dimension S is only 3.58mm (which corresponds to 0.06λ, being λ the wavelength for a 50 Ω transmission line on MgO).

The $k=4$ Hilbert resonator was realized on a 10x10mm^2 double sided 700nm YBCO thin film on MgO. The resonator was measured at T=77K in a liquid nitrogen bath resulting a Q of about 30000. It is worth to mention that this value is in very good agreement with the value predicted by Momentum software [10]. Moreover, it is very similar to those reported in very recent papers for other compact resonators [4] and [5].

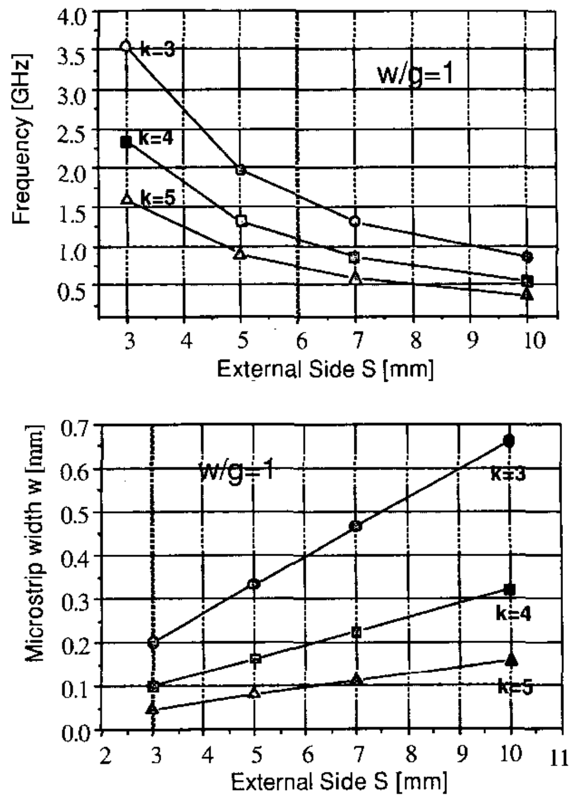


Fig. 2. a) fundamental resonant frequencies of hilbert resonators with k=3,4,5 and b) corresponding microstrip widths as a function of the external side s(k).

III. Filter Designs and Experimental Results

In this section we report the design and test of a four pole quasi-elliptic filter based on the $k=4$ Hilbert resonator (Fig.3). To do so, we slightly modify the layout of the resonator to fulfill the requirements to realize quasi-elliptic filters.

Quasi-elliptic filters need out-of-phase coupling between non-adjacent resonators [3]. In order to achieve out-of-phase couplings, the Hilbert resonator has been provided with a capacitive load by lengthening the ends of the resonator. The outline of the resonator is shown in Fig. 4a.

Following the procedure described in [11], a four pole quasi-elliptic filter can be implemented with a conventional cascaded-quadruplet topology. The couplings between the resonators are obtained by properly fixing the spacing between them. Input and output port (feed lines) are coupled to the first and last resonator, respectively.

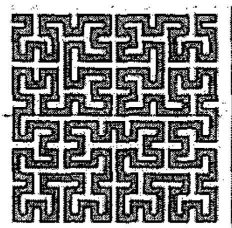


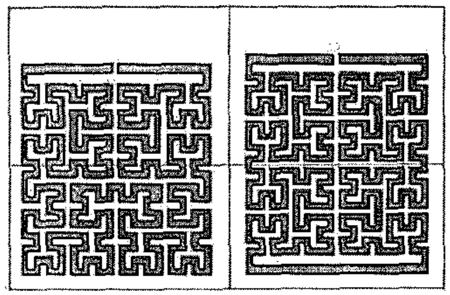
Fig. 3. Hilbert resonator with k=4.

According to our results, with this resonator it is not possible to obtain the desired values of Qext by coupling with capacitive gaps, or by direct connection to the strip (tapped line configuration) [11]. Capacitive coupling provides values of Qext larger than 200, suitable for filters with fractional bandwidths lower than 0.5%, whereas direct conection of the feedline with the resonator strip results in values of Qext lower than 20 (adequate for fractional bandwidths larger than 3%, i.e. 60 MHz at 2 GHz). This problem could be solved if the feedline could be connected closer to the point in the resonatro strip where the current is maximum. To achieve this, we have rearranged the Hilbert resonator as depicted in Fig. 4b. In this new configuration, the feed lines can be conected close to the current maximum, and Qext is determined by the separation between the contact point and the point where current is maximum.

Notice that the rearranging of Fig.4b consists on changing the orientation of two of the four component elements of $k=3$, in order to provide a double axial symmetric structure which makes accessible the point of maximum current.

Finally, using Fig.4b structure a quasi-elliptic filter with 20MHz bandwidth centred at 2.45GHz has been designed. The filter configuration is shown in Fig.5. The filter has been fabricated using a 10x10mm^2 double sided 700nm YBCO thin film deposited on 0.508μm thick MgO substrate. The basic resonator dimensions are 2.68mm x 2.28mm with a microstrip width of 90μm. The overall dimensions of the filter are 7.9mm x 6.2mm. The filter has been measured in a liquid nitrogen bath at P_{in}=0dBm. A slight shift in the central frequency has been observed between the simulated (f_0=2.45GHz) and the measured f_0=2.438GHz) responses. Fig. 6 compares the measured response - without tuning- with a shifted version of the simulated response. In Fig.6a, we see the agreement in bandwidth and transmission zero position between measured (continuous line) and simulated (dashed line) response.

a)



b)

Fig. 4. a) Hilbert resonator with capacitive load b) Hilbert resonator arrangement with accessible point of maximum current.

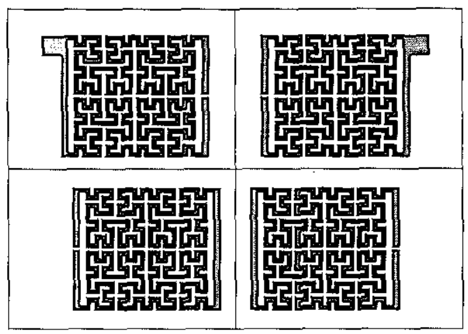


Fig. 5. Quasi-elliptic four pole filter.

The measured 3dB bandwidth (Fig.6b) differs 1MHz of the one expected from simulations. Fig.6b also shows a 0.2dB minimum insertion losses and a ripple distortion of 0.6dB, which corresponds to the 8dB measured return losses. These discrepances between measured and simulated in-band response might be attributed to detunning which, in turn, reduces the steepness at the upper bound of the passband and worsens the symmetry of the frequency response. The detunning may come from many parasitic effects, such as unwanted coupling, deviation in thickness substrate, tolerance in the fabrication process and so on [11].

As a last point, it is worthwhile to mention the absence of the second harmonic peak $2f_0$ as a consequence of the capacitive load of each resonator. In practice, at $2f_0$, the length of the resonators are as long as one wavelength, thus their ends present charges with the same sign (i.e. all couplings are in-phase) reducing considerably the effects of the equivalent capacitance [12]. The data shown in Fig.6c confirm the absence of the second harmonic.

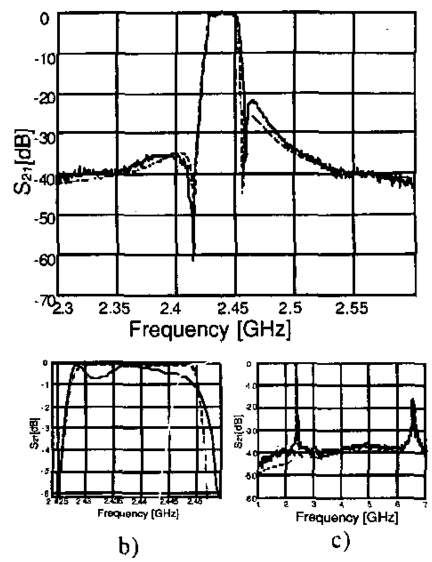


Fig. 6. a) Comparison between measured (continuous line) and simulation (dashed line) filter responses. In band (b) and large range (c) responses.

IV. CONCLUSION

In this work the miniaturization performance of a novel type of HTS microstrip resonator based on the Hilbert curves has been investigated. The fractal structure was analyzed considering different levels of iteration and putting in evidence the incidence of the different geometric parameters on the obtainable miniaturization level. The performance in terms of quality factor of a single Hilbert resonator with a side of 3.58mm and f_0=2GHz, has been measured at 77K showing a Q of about 30000. By using a commercial 10x10mm^2 YBCO thin film on MgO, a four pole quasi-elliptic filter centred close to 2.45GHz has been fabricated, showing the flexibility of this type of resonator. Agreement between between measurements and simulations has been observed. The measured insertion losses (0.1-0.2) dB confirm the good trade off between high performance and reduced dimensions.

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